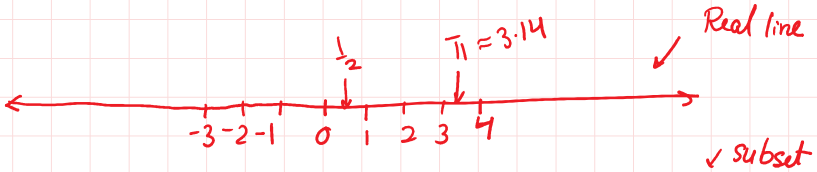
CH-1 Real Numbers

Real Numbers →



$\subset$ ✓ subset

N Natural Numbers → ~~0~~, 1, 2, 3, 4, ...

W Whole Number 0, 1, 2, 3, 4, ...

Z Integers ....., −3, −2, −1, 0, 1, 2, 3 ....

Q Rational Numbers → $\frac{p}{q}$ (where $q \neq 0$)

$N \subset W \subset Z \subset Q$

Ex $\frac{3}{10} = 0.3$

Terminating decimal expansion $q = 2^{\alpha} 5^{\beta}$

Non terminating and repeating decimal expansion $q \neq 2^{\alpha} 5^{\beta}$

$\frac{2}{3} = 0.666\cdots$
$= 0.\overline{6}$

T Irrational Numbers → Non terminating and non repeating decimal expansion

Ex

0.12112 1112 .....

$\pi = 3.14.....$

$\sqrt{2} = 1.41....$

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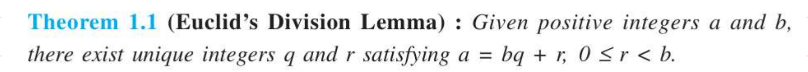
Q + T ⇒ Real Numbers

Rational number ↗ Q

Irrational Numbers ↑ T



Theorem 1.1 (Euclid's Division Lemma) : *Given positive integers a and b, there exist unique integers q and r satisfying $a = bq + r$, $0 \leq r < b$.*

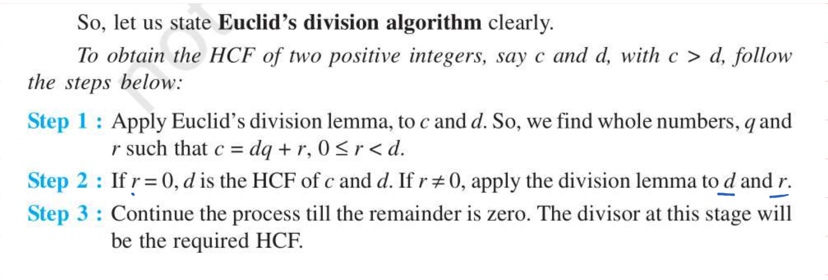
$\boxed{D = dq + r}$

$13 = 2(6) + 1$

2, 13

2 | 13 (6)
12
1

Divisor | Divident (Quotient)
⋮
Remainder



So, let us state **Euclid's division algorithm** clearly.

To obtain the HCF of two positive integers, say c and d, with c > d, follow the steps below:

Step 1 : Apply Euclid's division lemma, to c and d. So, we find whole numbers, q and r such that $c = dq + r$, $0 \le r < d$.

Step 2 : If $r = 0$, d is the HCF of c and d. If $r \ne 0$, apply the division lemma to $\underline{d}$ and $\underline{r}$.

Step 3 : Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

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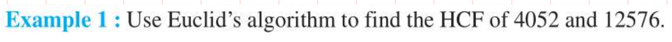

Example 1 : Use Euclid's algorithm to find the HCF of 4052 and 12576.

$12576 = \underline{4052} \times 3 + \underline{420}$

$4052 = \underline{420} \times 9 + \underline{272}$

$420 = 272 \times 1 + 148$

$272 = 148 \times 1 + 124$

$148 = \underline{124} \times 1 + \underline{24}$

$124 = \underline{24} \times 5 + \underline{4}$

$24 = \textcircled{4} \times 6 + \underline{0}$ —x—x—x—

$4052 \overline{)12576}$ = 3, 12156, 420

$420 \overline{)4052}$ = 9, 3780, 272

$420 \times 9 = 3780$

$272 \overline{)420}$ = 1, 272, 148

$148 \overline{)272}$ = 1, 148, 124

$124 \overline{)148}$ = 1, 124, 24

$24 \overline{)124}$ = 5, 120, 4

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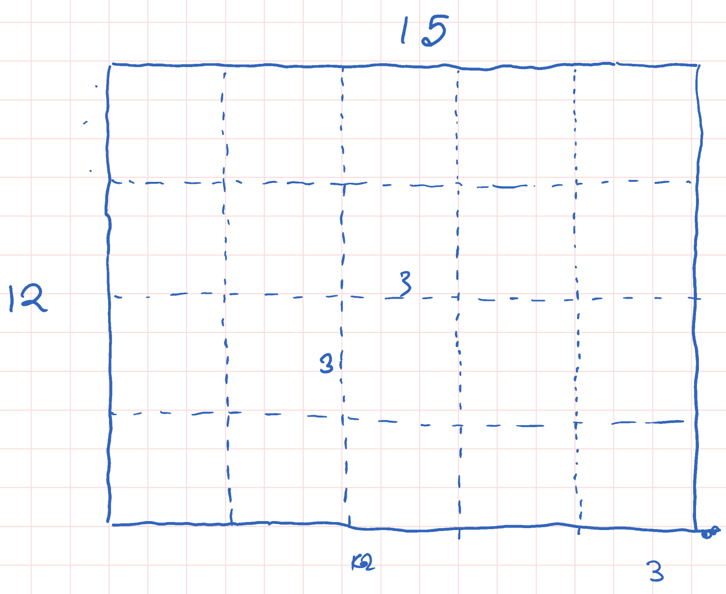
$\boxed{HCF = 4}$

Ex → Find HCF of 12 & 15



Using Prime Factorization. →

$12 = 2^2 \times 3^1 \times 5$

$15 = 2^0 \times 3^1 \times 5^1$

$HCF(12,15) = 2^0\ 3^1\ 5^0 = 3$

Using Euclids division Algorithm..

$15 = \underline{12} \times 1 + \underline{3}$

$12 = \textcircled{3} \times 4 + 0$

HCF = 3

LCM and HCF using prime factorization →

$15 = 3^1 \times 5^1 = 2^0 \times 3^1 \times 5^1$

$18 = 2^1 \times 3^2 = 2^1 \times 3^2 \times 5^0$

3 | 15
5 | 5
| 1

2 | 18
3 | 9
3 | 3
| 1

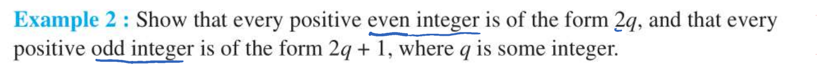
$\text{LCM}(15,18) = 2^1\, 3^2\, 5^1 \Rightarrow \boxed{90}$

$\text{HCF}(15,18) = 2^0\, 3^1\, 5^0 \Rightarrow \boxed{3}$

$\boxed{\text{LCM} \times \text{HCF} = \text{Product of Two numbers}}$ ☆ ☆

LHS $= 90 \times 3$
$= 270$

RHS $= 15 \times 18$
$= 270$ — x — x — x —



Example 2 : Show that every positive even integer is of the form $2q$, and that every positive odd integer is of the form $2q + 1$, where q is some integer.

Let 'a' be any positive number, let our second number be '2'

$a = bq + r$, where $r < b$

$a = 2q + r$, where $r < 2$

$\swarrow$ $\searrow$

$a = 2q$

$a = 2q + 1$

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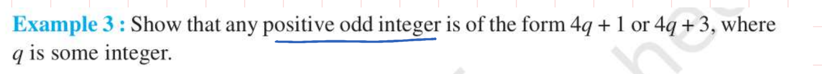
Ex

$8 = 2(4)$

$13 = 2(6) + 1$

$9 = 2(4) + 1$



Example 3 : Show that any positive odd integer is of the form $4q + 1$ or $4q + 3$, where q is some integer.

Let 'a' be any positive number and let $b = 4$

$a = bq + r$, where $r < b$

$a = 4q + r$, where $r < 4$

$r = 0$	$r = 1$	$r = 2$	$r = 3$
$a = 4q$	$a = 4q + 1$	$a = 4q + 2$	$a = 4q + 3$

$r=0$	$r=1$	$r=2$	$r=3$
$a=4q$	$a=4q+1$	$a=4q+2$	$a=4q+3$
Divisible by 2 Even number		Divisible by 2 Even number	

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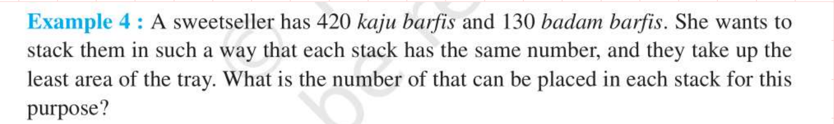
Any odd positive integer can be written as $4q+1$ or $4q+3$

$9 = 4(2) + 1$

$$\begin{array}{r|l} & 2 \\ 4 & 9 \\ & 8 \\ \hline & 1 \end{array}$$

$23 = 4(5) + 3$

$$\begin{array}{r|l} & 5 \\ 4 & 23 \\ & 20 \\ \hline & 3 \end{array}$$



Example 4 : A sweetseller has 420 *kaju barfis* and 130 *badam barfis*. She wants to stack them in such a way that each stack has the same number, and they take up the least area of the tray. What is the number of that can be placed in each stack for this purpose?

HCF (420 & 130)

$420 = 130 \times 3 + 30$

$130 = 30 \times 4 + 10$

$30 = 10 \times 3 + 0$

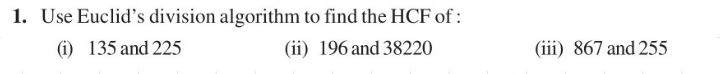
$$\boxed{HCF = 10}$$

$$\begin{array}{r|l} & 3 \\ 130 & 420 \\ & 390 \\ \hline & 30 \end{array}$$

$$\begin{array}{r|l} & 4 \\ 30 & 130 \\ & 120 \\ \hline & 10 \end{array}$$

$$\begin{array}{r|l} & 3 \\ 10 & 30 \\ & 30 \\ \hline & 0 \end{array}$$

Excercise 1.1



1. Use Euclid's division algorithm to find the HCF of :

(i) 135 and 225 (ii) 196 and 38220 (iii) 867 and 255

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(i) HCF (135,225)

$225 = 135 \times 1 + 90$

$$\begin{array}{r|l} & 1 \\ 135 & 225 \\ & 135 \\ \hline & 90 \end{array}$$

Applying Euclids divison Lemma on 135 & 90

$135 = 90 \times 1 + 45$

$$\begin{array}{r|l} & 1 \\ 90 & 135 \\ & 90 \\ \hline & 45 \end{array}$$

135 = 90 × 1 + 45

90
45

Applying Euclids division lemma on 90 & 45

2
45) 90
90
0

$90 = 45 \times 2 + 0$

HCF = 45

— x — x — x —

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(ii) HCF (196, 38820)

Applying EDL on 196 & 38820

1960
196
1874

19
196) 38820
196
1922
1874
148

$38820 = 196 \times 19 + 148$

Apply EDL on 196 & 148

$196 = 148 \times 1 + 48$

1
148) 196
148
48

Apply EDL on 148 & 48

3
48) 148
144
4

$148 = 48 \times 3 + 4$

Apply EDL on 48 & 4

$48 = \textcircled{4} \times 12 + 0$

12
4) 48
4
8
8
0

HCF = 4

— x — x —

(ii) HCF (196, 38220)

~~38220 > 196~~

195
196) 38220
196
1862
1764
980
980
x

$38220 = 196 \times 195 + 0$

HCF = 196

(iii) 867 & 255

3
255) 867
765

2
102) 255
204

$867 = 255 \times 3 + 102$

$867 = 255 \times 3 + 102$

255 | 867
765
102

102 | 255
204
51

$255 = 102 \times 2 + 51$

2
51 | 102
102
0

$102 = 51 \times 2 + 0$

HCF = 51

2. Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.

Let 'a' be any ~~nd~~ number and $b =$ '6'

Using Euclids division Lemma

$a = 6q + r \quad , r < 6$

× $r = 0 \Rightarrow a = 6q$ × ⇒ Even number

$r = 1 \Rightarrow \boxed{a = 6q + 1}$

× $r = 2 \Rightarrow a = 6q + 2$ × ⇒ Even number

$r = 3 \Rightarrow \boxed{a = 6q + 3}$

× $r = 4 \Rightarrow a = 6q + 4$ × ⇒ Even number

$r = 5 \Rightarrow \boxed{a = 6q + 5}$

Any odd positive integer can be written as $6q+1$, $6q+3$ or $6q+5$

—x—x—x—

3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

⇒ Applying EDL on 616 and 32

19
32 | 616
32
296
288
8

$616 = 32 \times 19 + 8$

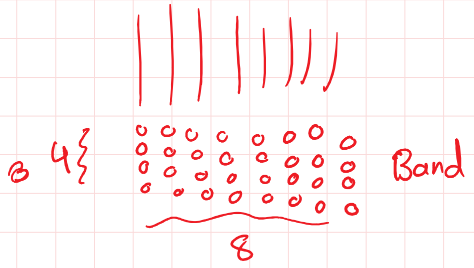
Applying EDL on 3~~2~~8 8

4
8 | 32
32
0

$32 = \boxed{8} \times 4 + 0$

HCF (616 & 32) = 8

Maximum columns in which they can



Maximum columns in which they can march = 8

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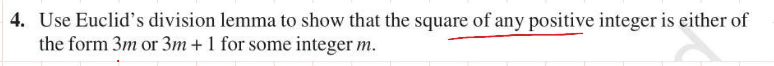

4. Use Euclid's division lemma to show that the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m.

Let 'a' be any number and $b = 3$

$q = bq + r$ $\quad r < b$

$a = 3q + r$ $\quad r < 3$

$a = 3q + 0$ —① $\longrightarrow a^2 = 9q^2 = 3(3q^2) = \boxed{3m}$

$a = 3q + 1$ —② $\longrightarrow a^2 = (3q+1)^2 = 9q^2 + 6q + 1$

$= 3(3q^2+2q) + 1$

$= \boxed{3m+1}$

$a = 3q + 2$ —③ $\longrightarrow a^2 = (3q+2)^2 = 9q^2 + 12q + 4$

$= 9q^2 + 12q + 3 + 1$

$= 3(3q^2+4q+1) + 1$

$= \boxed{3m+1}$

Square of any positive number is of kind $3m$ or $3m+1$

— x — x — x —

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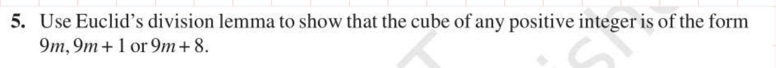
$4^2 = 16 = 3(5)+1$

$5^2 = 25 = 3(8)+1$

$6^2 = 36 = 3(12)$



5. Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

Let a be any number and $b = 3$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$a = bq + r$, $r < b$

$a = 3q + r$ $\quad r < 3$

$a = 3q \Longrightarrow a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$

$a = 3q + 1 \Longrightarrow a^3 = (3q+1)^3 = 27q^3 + 27q^2 + 9q + 1$

$= 9(3q^3 + 3q^2 + q) + 1$

$a = 3q + 2 \Longrightarrow$

$a = 3q + 1 \Rightarrow a^3 = (3q+1)^3 = 27q^3 + 27q^2 + 9q + 1$

$= 9(3q^3 + 3q^2 + q) + 1$

$= 9m + 1$

$a = 3q + 2 \Rightarrow$

$a^3 = (3q+2)^3 = (3q)^3 + 3(3q)^2(2) + 3(3q)(2)^2 + (2)^3$

$= 27q^3 + 54q^2 + 36q + 8$

$= 9(3q^3 + 6q^2 + 4) + 8$

$= 9m + 8$

Cube of any positive number is of the type

$9m, 9m+1$ or $9m+8$

Ex

$2^3 = 8 \Rightarrow 9(0) + 8$

$3^3 = 27 = 9(3)$

$4^3 = 64 = 9(7) + 1$

— x — x — x — x —

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